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AN ANALYSIS OF A MACROECONOMETRIC MODEL WITH RATIONAL EXPECTATIONS

IN THE BOND AND STOCK MARKETS

Ray C. Fair

August 17, 1977

AN ANALYSIS OF A MACROECONOMETRIC MODEL WITH RATIONAL EXPECTATIONS
IN THE BOND AND STOCK MARKETS*

by

Ray C. Fair**

I. Introduction

Accounting for expectational effects in macroeconomic models is a difficult problem. The standard procedure in econometric work in dealing with this problem, especially in the construction of large-scale macroeconomic models, is to use current and lagged values as "proxies" for expected future values. An alternative procedure, which has received increased attention lately in work with theoretical and small-scale empirical models, is to assume that expectations are rational.¹

For any given variable in a model, there are two main questions that can be asked about the expectations of its future values: (1) which expectational assumption best approximates the way that the expectations are actually formed, and (2) how sensitive are the properties of the model to alternative expectational assumptions? If for a particular variable the answer to the second question is that the properties are not very sen-

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¹For one class of models with rational expectations, see Lucas [7], Sargent [9], Sargent and Wallace [11], Barro [2], and Sargent [10]. See also Fair [5] for a criticism of this class of models. For an example of the use of the assumption of rational expectations in a small-scale empirical model (the St. Louis model), see Anderson [1].

sitive to alternative assumptions, then the answer to the first question for this variable is, of course, of less concern than otherwise.

The purpose of this paper is to provide an answer to the second question for a particular model and a particular set of variables. The model is the one described in Fair [4], and the variables are those that pertain to the bond and stock markets. Three "versions" of the model are considered: the original version in [4] and two modified versions. The original version, which will be called Model 1, does not have rational expectations in the bond and stock markets. There are two term-structure equations and one stock-price equation in the model, and in these three equations current and lagged values are used as proxies for expected future values. In the first modified version, which will be called Model 2, the two term-structure equations are replaced with a specification that is consistent with the existence of rational expectations in the bond market.² The second modified version, which will be called Model 3, is the same as Model 2 except that the stock-price equation is replaced with a specification that is consistent with the existence of rational expectations in the stock market. In Model 3, therefore, there are rational expectations in both the bond and stock markets. In the work below, the effects of fiscal-policy and monetary-policy actions in the original version are compared to those in the two modified versions.

With some qualifications, the main conclusion of this exercise is that fiscal-policy and monetary-policy actions are about half as effective

²The idea of replacing term-structure equations in macroeconometric models with a specification that is consistent with the existence of rational expectations in the bond market is contained in Poole [8], pp. 477-478. Poole (p. 478) questioned the computational feasibility of this procedure for large-scale models, but, as discussed below, this procedure was in fact computationally feasible for the model used in this study.

when there are rational expectations in the bond and stock markets than when there are not. For example, an expansionary fiscal policy leads to higher future short-term interest rates, and in the rational expectations versions of the model, this information gets incorporated immediately into the long-term rates. The response of the long-term rates to policy changes is thus faster in the rational expectations versions, which then offsets more than otherwise the effects of the policy changes.

In the numerical solution of the rational expectations versions of the model, future predicted values affect current predicted values, and so a more complicated iterative process must be used to solve the model in these cases. Some of the following discussion is thus concerned with explaining how the model was solved in the rational expectations cases.

It should be stressed at the outset that Model 3 is not a model in which all expectations are rational. The original version of the model (Model 1) is based on a theoretical model [3] in which individual agents engage in maximizing behavior, but do not know the complete model and so do not have rational expectations.³ Since Model 3 differs from the original version only with respect to the bond and stock markets, any non-rational expectations outside of the bond and stock markets that are implicit in the original version are also implicit in Model 3. If rational

³For the class of models mentioned in footnote 1, individual agents have rational expectations, but their decisions are not derived from the assumption of maximizing behavior. One way of characterizing the difference between this class of models and the basic theoretical model in [3] is thus to note that in the former there are non-maximizing agents with rational expectations, whereas in the latter there are maximizing agents without rational expectations. The assumption of non-maximizing agents with rational expectations is criticized in Fair [5] on the grounds that it does not seem plausible for agents to be rational with respect to their formation of expectations but irrational otherwise.

expectations exist at all, it seems more likely that they exist in the bond and stock markets than, say, in the labor and goods markets, and so for purposes of this paper it was decided to concentrate on the bond and stock markets. In future work it may be of interest to consider rational expectations in the labor and goods markets as well, although within the context of the present model this would be a much more difficult problem.⁴

A few of the features of the original version of the model are reviewed in Section II, and then the modifications of it are considered in Section III. The experiments that were performed using the three versions are described in Section IV, and the results of the experiments are presented and discussed in Section V. Section VI contains a summary of the main conclusions of this study and a suggestion for a possible future test of the assumption of rational expectations in the bond and stock markets.

⁴ Although the basic model in [3] is one in which there are maximizing agents without rational expectations, there is a special case of the model (the "static-equilibrium" version) in which there are maximizing agents with rational expectations. In this version all expectations are rational, and so in future work it might be of interest to construct an empirical model based on it. Any empirical model of this kind is likely to differ considerably from the three models considered in this study. See [3], Chapter 8, and [5] for further discussion of the static-equilibrium version.

II. A Brief Review of Interest-Rate and Wealth Effects in the Original Version

The econometric model in [4] consists of 84 equations, 26 of which are stochastic. There are five sectors (household, firm, financial, foreign, and government) and five categories of financial securities (demand deposits and currency, bank reserves, member bank borrowing from the Federal Reserve, gold and foreign exchange, and an "all other" category). Since the model is described in detail in [4], no extensive discussion of it will be presented here. It will be useful for purposes of the following analysis, however, to review briefly the interest-rate and wealth effects in the model.

There are three endogenous interest rates in the model: the three-month treasury bill rate (r), a Aaa corporate bond rate (RA), and a mortgage rate (RM).⁵ RM is an explanatory variable in three of the four consumption equations and in one of the three labor-supply equations; RA is an explanatory variable in the two interest-payment equations, in the stock-price equation, and in the main price equation of the model; and r is an explanatory variable in two of the consumption equations, in the two demand-for-money equations, in an equation explaining member bank borrowing from the Federal Reserve, and in one of the interest-payment equations. In addition, there is a loan-constraint variable in the model that is a function of r , and this variable is an explanatory variable in one of the consumption equations and in an equation explaining the dividend payments of the firm sector. r is also an explanatory variable in the term-

⁵In terms of the notation in [4], $r = RBILL$, $RA = RAAA$, and $RM = RMORT$.

structure equations for RA and RM.

The variable explained by the stock-price equation is CG, the value of capital gains (-) or losses (-) on corporate stocks held by the household sector. CG is part of the definition of ΔA in the model, where A is the value of securities (other than demand deposits and currency) held by the household sector. By definition, $\Delta A = -\Delta DDH + SAVH + CG$, where DDH is the value of demand deposits and currency held by the household sector and SAVH is the financial saving of the household sector. A lagged one quarter is an explanatory variable in three of the consumption equations and in one of the labor-supply equations.

If the amount of government securities outstanding (VBG) is taken to be exogenous, then r is implicitly determined in the model. In this case there is no equation in which r appears naturally as a left-hand-side variable. The feature of the model that allows r to be determined in this way is the fact that the model is closed with respect to the flow of funds in the system. Any financial saving or dissaving of a sector in a period results in the change in at least one of its assets or liabilities, and a financial asset of one sector is a corresponding liability of some other sector. The solution value of r each period is the value that leads to the equality between assets and liabilities being satisfied.

The treatment of VBG as exogenous means that the behavior of the Federal Reserve (henceforth called the "Fed") is treated as exogenous. In other words, the behavior of the Fed is assumed not to be influenced by the state of the economy. In a recent study [6], I have estimated an equation explaining Fed behavior, an equation in which r is the left-hand-side variable. If this equation is added to the model, then the behavior of the Fed is endogenous. In this case VBG must be taken to be

endogenous.⁶ Given the value of r that satisfies the Fed behavioral equation, the solution value of VBG each period is the value that leads to the equality between assets and liabilities being satisfied.

In summary, then, r affects directly RA, RM, the loan-constraint variable, two consumption variables, two demand-for-money variables, member bank borrowing from the Fed, and one interest-payment variable. It affects indirectly through the loan-constraint variable one consumption variable and one dividend variable. It affects indirectly through RA and RM three consumption variables, one labor-supply variable, two interest-payment variables, the main price variable in the model,⁷ and CG. In addition, CG affects A, which affects with a lag of one quarter three consumption variables and one labor-supply variable. These latter effects are wealth effects on the household sector. r in turn is affected by all the other variables in the model when it is implicitly determined (VBG exogenous). When the behavior of the Fed is endogenous (VBG endogenous), then r is determined according to the Fed behavioral equation.

III. The Modifications of the Original Version

The Term-Structure Equations in the Original Version

The two long-term rates, RA and RM, are functions in [4] of the current value of r and of lagged values of r and the inflation rate. In the

⁶This sentence is not quite right in that one of the other two policy variables of the Fed in the model (the discount rate and the reserve requirement ratio) could be taken to be endogenous instead of VBG. In this paper, however, these two policy variables are always assumed to be exogenous.

⁷The main price variable (PF) affects the level of sales, which affects the level of production, which affects the levels of investment and employment. r thus has an indirect effect on investment and employment through its effect on PF. See Chapter 5 in [4] for further discussion of this point.

theoretical model in [3], upon which the econometric model in [4] is based, the long-term bond rate is a function of the current and expected future values of r , and in the empirical work the current value of r and the lagged values of r and the inflation rate were used as proxies for the expected future values of r . The estimated equations for RA and RM are:⁸

$$\begin{aligned}
 (1) \quad \log RA_t &= 0.0695 + 0.915 \log RA_{t-1} + 0.1767 \log r_t - 0.1867 \log r_{t-1} \\
 &\quad (4.10) \quad (46.86) \quad (3.70) \quad (3.07) \\
 &\quad + 0.0636 \log r_{t-2} + 1.27 \left(\frac{1}{2} \Delta \log PX_{t-1} + \frac{1}{3} \Delta \log PX_{t-2} + \frac{1}{6} \Delta \log PX_{t-3} \right), \\
 &\quad (2.34) \quad (2.23) \\
 R^2 &= 0.996, SE = 0.0223, DW = 1.80.
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad \log RM_t &= 0.1965 + 0.852 \log RM_{t-1} + 0.0297 \log r_t + 0.0854 \log r_{t-1} \\
 &\quad (3.87) \quad (24.33) \quad (0.70) \quad (1.56) \\
 &\quad - 0.1138 \log r_{t-2} + 0.0551 \log r_{t-3} + 1.59 \left(\frac{1}{2} \Delta \log PX_{t-1} + \right. \\
 &\quad (3.00) \quad (2.38) \quad (1.87) \\
 &\quad \left. \frac{1}{3} \Delta \log PX_{t-2} + \frac{1}{6} \Delta \log PX_{t-3} \right), \\
 \hat{\rho} &= 0.247, R^2 = 0.988, SE = 0.0254, DW = 1.93. \\
 &\quad (2.41)
 \end{aligned}$$

The last term in equations (1) and (2) is a weighted average of the rates of inflation in the past three quarters, with weights of $1/2$, $1/3$, and $1/6$. PX is one of the price deflators in the model. Note that each equation includes as explanatory variables both the lagged dependent variable and the current and lagged values of r , which implies a fairly complicated lag structure of r on both the long term rates.

⁸For purposes of the work in this paper the model was re-estimated through 1976 II using the revised national-income-accounts data. The estimates in equations (1) and (2) thus differ somewhat from those presented in [4] (Table 2-3). Likewise, the estimates presented in equation (3) below for CG differ somewhat from those in [4]. The estimated version of the model used in this study is the same as the one used for the results in [6]. $\hat{\rho}$ in equation (2) is the estimate of the first-order serial correlation coefficient. t -statistics in absolute value are in parentheses. The equations were estimated by two-stage least squares, as described in Chapter 3 in [4].

The Replacements for the Term-Structure Equations

Let ${}_t r_{t+1}^e$ denote the expected one-period rate of return for period $t+1$, the expectation being made at the beginning of period t . If RA_t is the long-term rate in period t on an n -period security, then according to the expectations theory of the term structure of interest rates, the following equation must hold:

$$(1)' \quad (1+RA_t)^n = (1+{}_t r_t^e)(1+{}_t r_{t+1}^e) \dots (1+{}_t r_{t+n-1}^e).$$

In other words, the return from holding an n -period security must equal the expected return from holding a series of one-period securities over the n periods. Equation (1)' abstracts from considerations of transactions costs, preferred habitats, and differential tax treatments.

Equation (1)' is an implication of the expectations theory of the term structure of interest rates. It holds regardless of how expectations of the future values of r are formed. When considered by itself, equation (1) is thus consistent with equation (1)' in the sense that the current value of r and the lagged values of r and the inflation rate in (1) are proxying for the expected future values of r in (1)'. When considered as part of the overall model, however, equation (1) is not consistent with equation (1)' if expectations of the future values of r are rational. This is because in simulations of the model the predicted values of RA_t , r_t , r_{t+1} , ..., r_{t+n-1} do not in general satisfy (1)'.

It is possible to replace equation (1) by equation (1)' in the model, where the values of ${}_t r_t^e$, ${}_t r_{t+1}^e$, ..., ${}_t r_{t+n-1}^e$ in (1)' are taken to be the values predicted by the model, and this is what was done for the two modified versions. Likewise, an equation like (1)' for RM_t was used to replace equation (2). These two changes make the model more difficult to solve computationally, since future predicted values now affect present predicted values, but for the work in this study it was computationally feasible to solve the model in this case.⁹

With respect to the determination of the long-term interest rates, the two modified versions of the model thus consist of the replacement of equation (1) by equation (1)' and of equation (2) by an equation like (1)' with RM_t in place of RA_t . These versions are consistent with the existence of rational expectations in the bond market: the expected future one-period rates that are implicit in the current long-term rates are the same as the future one-period rates predicted by the model.¹⁰ It should be noted, since the structure of the model is assumed to be known for the calculations, that the amount of information used in generating the future predicted values of r is greater than the amount contained only in lagged values.

⁹As mentioned in Poole [8], pp. 477-478, in order to solve a model in this case, one must iterate on solution paths. For, say, a 40-quarter problem, one first solves the model for the 40 quarters using either guessed values of RA and RM for each of the 40 quarters or else values predicted by the term-structure equations. New values of RA and RM are then computed using equation (1)' and the predicted values of r . The model is then resolved for the 40 quarters using these new values of RA and RM . New values of RA and RM are then computed using the new predicted values of r , and the model is solved again. There is no guarantee that this process will converge, but for the work in this study it did converge after some damping of some of the solution values. It took an average of about 15 iterations for this process to converge, which means that the model is about 15 times more expensive to solve in this case than it is in the regular case. It should also be noted that for the results in this study, values of r beyond the end of the data period were needed. The procedure that was followed to construct these values is discussed in Section IV.

¹⁰One somewhat subtle point should be noted here. When a nonlinear model is solved by setting the error terms equal to their expected values, the solution values of the endogenous variables are not in general equal to their expected values. The proper way to solve nonlinear models is by means of stochastic simulation, but because of the expense, this is rarely done. Since no stochastic simulation was done in this study, the predicted values of the future one-period rates generated by the model are not exactly equal to their expected values. For ease of exposition, however, no distinction will be made in the text between predicted and expected values.

The Stock-Price Equation in the Original Version

As mentioned above, CG in the model in [4] is the value of capital gains or losses on corporate stocks held by the household sector. It is a function in [4] of the current change in RA and of a weighted average of the current and past changes in after-tax cash flow. In the theoretical model in [3], the aggregate value of stocks is determined as the present discounted value of expected future after-tax cash flow,¹¹ the discount rates being the expected future values of r . The theoretical model thus implies that CG should be a function of changes in expected future after-tax cash flow and of changes in the expected future values of r . In the empirical work, the current change in RA was used as a proxy for changes in the expected future values of r , and the weighted average of the current and past changes in after-tax cash flow was used as a proxy for changes in expected future after-tax cash flow. The estimated equation for CG is:

$$(3) \quad CG_t = 13.19 - 124.3 \Delta RA_t + 9.824 \left(\frac{1}{2} \Delta \pi_t + \frac{1}{3} \Delta \pi_{t-1} + \frac{1}{6} \Delta \pi_{t-2} \right),$$

(2.53) (3.19) (2.12)

$$R^2 = 0.212, SE = 44.02, DW = 2.33.$$

The last term in equation (3) is a weighted average of the change in π for the current and past two quarters, with weights of 1/2, 1/3, and 1/6, where π is the value of after-tax cash flow of the firm sector.¹²

The Replacement for the Stock-Price Equation

Let SP_t denote the value of corporate stocks held by the household

¹¹In the theoretical model all after-tax cash flow is paid out in dividends, and so the value of stocks in this model is merely the present discounted value of expected future dividends. In the empirical model, as in the real world, some after-tax cash flow is kept as retained earnings.

¹²In terms of the notation in [4], $\pi = CF - TAXF$.

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sector at the end of period t . In the model in [4], SP_t is part of A_t , the total value of securities (except demand deposits and currency) held by the household sector. Although SP_t was not used as a separate variable for the work in [4], data on it were collected in the process of constructing a series for CG_t . By definition:

$$(4) \quad CG_t = SP_t - SP_{t-1}.$$

Let ${}_t\Pi_{t+i}^e$ denote the expected value of after-tax cash flow for period $t+i$, the expectation being made at the beginning of period t . If, as in the theoretical model, the value of stocks is the present discounted value of expected future after-tax cash flow, then

$$(5) \quad SP_{t-1} = \frac{{}_t\Pi_t^e}{(1+{}_tr_t^e)} + \frac{{}_t\Pi_{t+1}^e}{(1+{}_tr_t^e)(1+{}_tr_{t+1}^e)} + \dots + \frac{{}_t\Pi_{t+T}^e}{(1+{}_tr_t^e)(1+{}_tr_{t+1}^e)\dots(1+{}_tr_{t+T}^e)}$$

where T is large enough to make the last term in (5) negligible. An equation like (5) also holds, of course, for SP_t , with $t+1$ replacing t :

$$(6) \quad SP_t = \frac{{}_{t+1}\Pi_{t+1}^e}{(1+{}_{t+1}r_{t+1}^e)} + \frac{{}_{t+1}\Pi_{t+2}^e}{(1+{}_{t+1}r_{t+1}^e)(1+{}_{t+1}r_{t+2}^e)} + \dots + \frac{{}_{t+1}\Pi_{t+T+1}^e}{(1+{}_{t+1}r_{t+1}^e)(1+{}_{t+1}r_{t+2}^e)\dots(1+{}_{t+1}r_{t+T+1}^e)}.$$

Equations (5) and (6) also abstract from considerations of transactions costs and differential tax treatments.

Equations (5) and (6) are an implication of the standard theory of stock-price determination. They hold regardless of how expectations of the future values are formed. When considered by itself, equation (3) is thus consistent with equations (4)-(6) in the sense that ΔRA_t and the weighted average term in (3) are proxying for the changes in expected future interest rates and after-tax cash flow that are implicit in (4). Again, however, when equation (3) is considered as part of the overall model, it is not

consistent with equations (4)-(6) if expectations of the future values are rational. This is because in simulations of the model the predicted values of CG_t , Π_t , ..., Π_{t+T+1} , r_t , ..., r_{t+T+1} do not in general satisfy (4)-(6).

It is possible to replace equation (3) by equations (4)-(6) in the model, where the expected values in (5) and (6) are taken to be the values predicted by the model, and this is what was done for Model 3. This change also makes the model more difficult to solve, since future predicted values again affect present predicted values. The additional computations needed for this change over those already needed for the term-structure change, however, turned out to be small.

Model 3 is consistent with the existence of rational expectations in the stock market: the expected values in (5) and (6) are the same as the values predicted by the model.¹³ Also, because r is used as the discount rate in (5) and (6), the expected rate of return on stocks is the same as the expected rate of return on bonds. In other words, there are no arbitrage opportunities in Model 3 between bonds and stocks, just as there are no arbitrage opportunities in either Model 2 or 3 between bonds of different maturities.

IV. The Experiments

Two basic experiments were performed for each of the three models: the first a fiscal-policy action and the second a monetary-policy action. For the first experiment, the real value of goods purchased by the government (denoted as XG in the model) was increased by 1.25 billion dollars

¹³ With respect to equations (5) and (6), it should be noted that in any given solution of the model, ${}_t r_{t+i}^e = {}_{t+1} r_{t+i}^e$ for all i , since the expected value of r for period $t+i$ is merely the value predicted by the model for that period.

beginning in 1971 I, a quarter that is at or near the bottom of a contraction. The behavior of the Fed was assumed to be endogenous for this experiment: the equation explaining Fed behavior (the equation with r on the left hand side) was added to the model, and VBG, the amount of government securities outstanding, was taken to be endogenous. For the second experiment, VBG was decreased by 1.25 billion dollars beginning in 1971 I. In this case the behavior of the Fed is not endogenous, so the equation explaining Fed behavior is not included in the model. No fiscal-policy variables were changed for the second experiment, which means that the behavior of the fiscal authorities was assumed to be exogenous. It should thus be noted that there is an important asymmetry between the two experiments: monetary policy is endogenous for the fiscal-policy action, but fiscal policy is exogenous for the monetary-policy action.

The experiments for Model 1 were performed as follows. The residuals obtained in the process of estimating each equation of the model were first added to the equations. This means that when the model is simulated using the actual values of all exogenous variables, the predicted values of all endogenous variables are equal to their actual values. In other words, a perfect tracking solution is obtained. These residuals were then used for all the experiments for Model 1. The simulations were dynamic, 12-quarter simulations for the period 1971 I - 1973 IV. The results of changing XG are presented in Table 1, and the results of changing VBG are presented in Table 2. The results in Table 1 for Model 1 are the same as the results presented in Table 1 in [6] for the endogenous Fed case.

The experiments for Models 2 and 3 require a little more explanation. For Model 2 they were performed as follows. First, n in equation (1)' was taken to be 32. In other words, RA was assumed to be the rate

on an 8-year bond. Second, the values of r beyond the end of the data period (1976 II), which are needed in the calculations of RA, were all assumed to be equal to the average of the last 8 observed values of r (i.e., to the average of the values of r for 1974 III-1976 II). Third, equation (1)' was used to compute predicted values of RA over the sample period, using $n = 32$, the actual values of r within the data period, and the constructed values of r beyond the end of the data period. The differences between the actual and predicted values of RA were then computed, and these residuals were added to equation (1)'.

Equation (1)' with RM in place of RA was used to compute predicted values of RM over the sample period. The same assumptions about n and r were used for RM as were used for RA, which means that for each period the predicted values of RM and RA are the same. The residuals are different, however, since RM does not in general equal RA in the actual data. The residuals for RM were added to the RM version of equation (1)'. Adding the RA residuals to equation (1)' and the RM residuals to the equivalent equation for RM means that a perfect tracking solution is obtained for Model 2 when the actual values of all exogenous variables are used. Model 2 is thus on a par with Model 1 with respect to the starting position for each experiment.

The simulations for Model 2 were dynamic, 22-quarter simulations for the period 1971 I-1976 II. Since future predicted values affect current predicted values for Model 2, the simulation period was taken to be as long as the data would allow, given the starting point. Note, however, that only results for the first 12 quarters of the simulation period are presented in Tables 1 and 2 for Model 2. It should also be noted that the values of r beyond the end of the data period that were used for the

simulations were taken to be the average of the last 8 predicted values of r , not the last 8 actual values. Only for the perfect tracking solution are the last 8 predicted values equal to the last 8 actual values.

For the experiments for Model 3, T in equations (5) and (6) was taken to be 80. In other words, the horizon for determining stock prices was assumed to be 20 years. Also, as with r , the values of π beyond the end of the data period were all assumed to be equal to the average of the last 8 observed values. Equation (6) was first used to generate predicted values of SP over the sample period, using $T = 80$, the actual values of π and r within the data period, and the constructed values of π and r beyond the end of the data period. The same assumption regarding the values of r beyond the end of the data period was used for Model 3 as was used for Model 2. The differences between the actual and predicted values of SP were then computed, and these residuals were added to equations (5) and (6). This procedure also means that a perfect tracking solution is obtained for Model 3 when the actual values of all exogenous variables are used. The simulations for Model 3 were also dynamic, 22-quarter simulations for the period 1971 I-1976 II. Results for the first 12 quarters of this period are presented in Tables 1 and 2.

One further point about the solution for Model 3 should be noted. If, say, the first quarter of the simulation period is quarter t , then for Models 1 and 2, SP_{t-1} , the value of stocks at the beginning of quarter t , is predetermined. For Model 3, however, SP_{t-1} is endogenous. It is determined according to equation (5), where the future values of π and r in the equation are the values predicted by the model. This means that CG_{t-1} is also endogenous for Model 3, since it equals $SP_{t-1} - SP_{t-2}$. Consequently, values of ΔCG are presented in Tables 1 and 2 for quarter $t-1$

as well as for the other 12 quarters for Model 3.

Two other experiments were also performed in this study for the three models. One of these was the same as the first experiment except that the beginning point was taken to be 1958 I rather than 1971 I. For Model 1 the end period was changed to 1960 II (12 quarters from the beginning), but for Models 2 and 3 the end period remained at 1976 II (74 quarters from the beginning). For this longer period, the results for Models 2 and 3 for, say, the first 12 quarters should be less sensitive to the assumption about the values of r and π beyond the end of the data period. It turned out, however, that these results were quite similar to the results for the shorter period of the first experiment. They are briefly discussed in the next section.

The other experiment was the same as the first experiment except that the behavior of the Fed was assumed to be different. Instead of behaving according to the estimated equation, the Fed was assumed to behave by keeping r unchanged each period from its historic value. In other words, VBG was adjusted each period to keep r unchanged from its historic value. In this case the behavior of the Fed is exogenous in the sense that its behavior with respect to the value of r each period is not a function of any endogenous variables in the model. This experiment is thus an example of a fiscal-policy action with an exogenous monetary policy. The results of it are briefly discussed in the next section. No experiments of a monetary-policy action with an endogenous fiscal policy were performed in this study.

V. The Results

The results of the two experiments for the three models are presented

in Tables 1 and 2. The effects on seven variables in the model are presented. Each number in the tables is the difference between the predicted value of the variable for the quarter and the actual value. Y is the key output variable in the model, and PF is the key price variable.

Consider the results in Table 1 first. The increase in XG led to an increase in output in all three models. The sum of the output increases over the 12 quarters was 16.81 for Model 1, 9.59 for Model 2, and 10.27 for Model 3. The increases in RA and RM for the first four quarters were much smaller for Model 1 than they were for the other two models. This is, of course, as expected. In Models 2 and 3, RA and RM adjust immediately to the higher future values of r induced by the increase in XG, whereas in Model 1 they adjust only to the current and lagged increases in r . Higher values of RA and RM have, other things being equal, a contractionary effect on the economy, and this is the primary reason for the lower values of Y for Models 2 and 3 compared to those for Model 1. RA also has, other things being equal, a positive effect on PF, and this is the main reason for the higher values of PF for Models 2 and 3 compared to those for Model 1.

It is interesting to note that the values of r are higher for Model 1 than they are for Models 2 and 3. The Fed behavioral equation that determines r is an equation in which the Fed "leans against the wind."¹⁴ In other words, the Fed responds over time to an expanding economy by raising r . Therefore, since the economy is less expansionary for Models 2 and 3 than it is for Model 1, the Fed raises r less in these two cases than it does in the Model 1 case.

The economy is slightly more expansionary for Model 3 than it is for

¹⁴The Fed behavior equation is presented and discussed in [6].

TABLE 1. FISCAL-POLICY RESULTS

Effects of a permanent increase in XG of 1.25 billion dollars beginning in quarter t (t = 1971 I).

- 1 = Model 1 (original version)
 2 = Model 2 (rational expectations in bond market)
 3 = Model 3 (rational expectations in bond and stock markets)

VARIABLES	QUARTERS												SUM 12
	t	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8	t+9	t+10	t+11	
<u>r (bill rate)</u>													
1	0.10	0.20	0.26	0.30	0.31	0.31	0.30	0.28	0.27	0.25	0.24	0.24	
2	0.08	0.15	0.18	0.19	0.19	0.19	0.18	0.18	0.17	0.16	0.16	0.16	
3	0.08	0.15	0.18	0.20	0.20	0.20	0.20	0.19	0.19	0.18	0.18	0.18	
<u>RA (bond rate)</u>													
1	0.03	0.06	0.07	0.11	0.14	0.15	0.16	0.17	0.18	0.18	0.18	0.19	
2	0.21	0.21	0.22	0.22	0.22	0.22	0.22	0.22	0.23	0.23	0.23	0.23	
3	0.23	0.24	0.24	0.25	0.25	0.25	0.25	0.26	0.26	0.26	0.27	0.27	
<u>RM (mortgage rate)</u>													
1	0.01	0.03	0.05	0.07	0.10	0.13	0.14	0.15	0.15	0.16	0.16	0.17	
2	0.21	0.21	0.22	0.22	0.22	0.22	0.22	0.22	0.23	0.23	0.23	0.23	
3	0.23	0.24	0.24	0.25	0.25	0.25	0.25	0.26	0.26	0.26	0.27	0.27	
<u>Y (real output)</u>													
1	1.24	1.77	2.04	2.12	1.95	1.72	1.48	1.25	1.03	0.84	0.71	0.66	16.81
2	0.98	1.12	1.11	1.08	0.97	0.86	0.77	0.68	0.58	0.49	0.46	0.49	9.59
3	0.93	1.10	1.12	1.12	1.03	0.94	0.86	0.77	0.67	0.58	0.56	0.59	10.27
<u>100 · PF (price deflator)</u>													
1	0.02	0.05	0.09	0.14	0.19	0.25	0.29	0.33	0.37	0.41	0.45	0.48	
2	0.07	0.13	0.18	0.23	0.27	0.31	0.34	0.37	0.39	0.42	0.45	0.47	
3	0.08	0.15	0.20	0.25	0.30	0.34	0.38	0.41	0.44	0.47	0.50	0.53	
<u>CG (capital gains or losses on stocks)</u>													
<u>t-1</u>													
1	0.00	-2.14	-3.34	-2.55	-6.19	-5.85	-1.74	-1.47	-0.48	-0.37	0.03	0.09	-0.77 -24.78
2	0.00	-24.40	-1.18	-1.04	-0.82	-0.35	-0.06	0.02	0.06	-0.07	-0.15	-0.14	-0.09 -28.22
3	-3.13	-0.34	-0.05	-0.01	-0.00	0.04	0.00	-0.03	-0.06	-0.08	-0.10	-0.15	-0.16 -4.07
<u>VBG (amount of government securities outstanding)</u>													
1	0.66	1.07	1.34	1.61	1.90	2.29	2.79	3.41	4.12	4.93	5.82	6.75	
2	0.67	1.29	1.90	2.58	3.29	4.06	4.87	5.72	6.64	7.61	8.62	9.66	
3	0.68	1.28	1.85	2.46	3.09	3.76	4.46	5.20	6.00	6.85	7.72	8.61	

Notes: Units of variables are: percentage points at an annual rate for r, RA, and RM;
 billions of 1972 dollars at a quarterly rate for XG and Y; 1972 = 1.0 for PF;
 billions of current dollars at a quarterly rate for CG; billions of current
 dollars for VBG.

TABLE 2. MONETARY-POLICY RESULTS

Effects of a permanent decrease in VBG of 1.25 billion dollars beginning in quarter t (t = 1971 I).

- 1 = Model 1 (original version)
 2 = Model 2 (rational expectations in bond market)
 3 = Model 3 (rational expectations in bond and stock markets)

VARIABLES

QUARTERS

SUM OVER
12 QUARTERS

	t	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8	t+9	t+10	t+11		
<u>r (bill rate)</u>														
1	-1.74	0.37	1.41	1.43	0.78	-0.05	-0.55	-0.61	-0.40	-0.13	0.12	0.25		
2	-1.35	-0.50	0.06	0.34	0.35	0.27	0.18	0.10	0.05	0.04	0.04	0.05		
3	-1.35	-0.52	0.04	0.33	0.36	0.30	0.22	0.13	0.08	0.06	0.05	0.05		
<u>RA (bond rate)</u>														
1	-0.73	0.21	0.12	0.19	0.16	-0.01	-0.07	-0.06	-0.03	-0.02	-0.01	0.01		
2	-0.02	0.04	0.05	0.05	0.04	0.03	0.02	0.01	0.01	0.01	0.01	0.01		
3	-0.02	0.04	0.06	0.05	0.04	0.03	0.01	0.01	0.01	0.00	0.00	0.00		
<u>RM (mortgage rate)</u>														
1	-0.14	-0.48	0.21	0.08	0.14	0.11	-0.00	-0.03	-0.02	-0.01	-0.02	-0.01		
2	-0.02	0.04	0.05	0.05	0.04	0.03	0.02	0.01	0.01	0.01	0.01	0.01		
3	-0.02	0.04	0.06	0.05	0.04	0.03	0.01	0.01	0.01	0.00	0.00	0.00		
<u>Y (real output)</u>														
1	0.77	1.98	1.39	0.44	-0.51	-0.85	-0.62	-0.19	0.14	0.27	0.26	0.15	3.23	
2	0.39	0.59	0.54	0.35	0.13	0.00	-0.06	-0.07	-0.06	-0.04	-0.02	-0.02	1.73	
3	0.39	0.57	0.54	0.36	0.15	0.02	-0.04	-0.06	-0.05	-0.03	-0.02	-0.01	1.82	
<u>100 · PF (price deflator)</u>														
1	-0.24	-0.10	-0.02	0.06	0.11	0.08	0.03	0.00	-0.00	0.00	0.01	0.02		
2	-0.00	0.02	0.04	0.05	0.06	0.06	0.06	0.05	0.05	0.05	0.04	0.04		
3	-0.00	0.02	0.04	0.05	0.06	0.06	0.06	0.05	0.05	0.04	0.04	0.04		
<u>CG (capital gains or losses on stocks)</u>														
	t-1													
1	0.00	92.54	-117.66	8.89	-11.96	2.57	23.66	10.47	0.13	-3.09	-2.37	-2.63	-2.70	-2.15
2	0.00	2.76	-7.30	-2.35	-0.44	1.19	1.95	1.53	0.93	0.45	0.22	0.07	0.04	-0.95
3	-2.00	-0.78	-0.21	0.15	0.34	0.33	0.21	0.11	0.04	0.02	0.02	0.03	0.04	-1.70
<u>VBG (amount of government securities outstanding)</u>														
1	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	
2	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	
3	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	

Notes: See Table 1.

Model 2, and this is easy to explain. For Model 2 there was a large capital loss on stocks in quarter t because of the increase in RA . For Model 3, however, the negative effects of the higher future values of r on the value of stocks were almost completely offset by the positive effects of higher future values of after-tax cash flow caused by the increase in economic activity. The capital loss incurred at the beginning of quarter t was 3.13 for Model 3, compared to the capital loss in quarter t of 24.40 for Model 2. Since capital losses have a negative effect on the economy through the wealth effect on the household sector, the economy was somewhat more expansionary for Model 3 than it was for Model 2. This difference is, however, much smaller than the difference between the results for Models 1 and 2. In other words, adding rational expectations in the bond market to the original version of the model makes more of a difference than does the further addition of rational expectations in the stock market. In this sense, wealth effects in the model are less important than interest-rate effects.

Consider now the monetary-policy experiment in Table 2. The experiment itself is not as expansionary as the experiment in Table 1, but the comparison across models in Table 2 is similar to that in Table 1. The economy is more expansionary for Model 1 than it is for Models 2 and 3, and it is slightly more expansionary for Model 3 than it is for Model 2. For Model 1 the decrease in VBG led to a large decrease in r in quarter t and then a bounce back again in quarter $t+1$. RA was affected in a similar way, which resulted in a large capital gain in quarter t and a large capital loss in quarter $t+1$. For Models 2 and 3, on the other hand, RA and RM were much less affected by the large initial change in r . RA and RM in fact changed very little for Models 2 and 3 because the long-run effect of the

change in VBG on r was fairly small.

Comparing the results for Models 2 and 3 in Table 2, after the first four quarters Model 2 has a cumulative capital loss of 7.33 compared to a cumulative capital loss of only 2.50 for Model 3. This is the main reason for the slightly more expansionary economy for Model 3. The differences between the results for the two models is, however, quite small.

In summary, then, the results in Tables 1 and 2 indicate that the long-run effects of fiscal-policy and monetary-policy actions on real output are a little over half as large if there are rational expectations in the bond and stock markets than if there are not. For the fiscal-policy experiment, the sum of the output increases over 12 quarters for Model 3 is 61.1 percent of that for Model 1 ($10.27/16.81$). The corresponding figure for the monetary-policy experiment is 56.3 percent ($1.82/3.23$). The results also indicate that the addition of rational expectations in the bond market to the model is quantitatively more important than is the addition of rational expectations in the stock market.

The results of the other two experiments are easy to summarize. For the experiment that began in 1958 I, the sum of the changes in Y over the first 12 quarters was 16.27 for Model 1 and 8.04 for Model 3. The Model 3 response was thus 49.4 percent of the Model 1 response ($8.04/16.27$), which is only slightly lower than the figure of 61.1 percent for the first experiment. The other results were also similar between the two experiments. It thus appears that the use in the first experiment of only 10 quarters beyond the basic 12-quarter prediction period for Models 2 and 3 is enough to capture most of the effects of the future predicted values on the present predicted values.

For the experiment in which r was kept unchanged each period from

its historic value, the results for Models 1 and 2 were nearly identical.

The sum of the changes in Y over the first 12 quarters was 27.73 for Model 1 and 28.07 for Model 2. For Model 2, RA and RM were completely unchanged, as is obvious from equation (1)', but for Model 1 they were slightly higher as a result of the inflation term in equations (1) and (2). The slightly higher values of RA and RM thus led to a slightly smaller output increase in Model 1 than in Model 2. This difference is, however, almost negligible. The main point of this example is that if the Fed keeps r unchanged, then it makes little difference if one uses a model with rational expectations in the bond market or a model with the more traditional term-structure equations.

Two further remarks about the results in Tables 1 and 2 should be made. The first concerns the implicit assumption for Models 2 and 3 that agents know the model, the future values of the exogenous variables, and the future values of the error terms. This assumption is less restrictive than it might appear to be at first glance, since the only key part of the assumption is that agents know the model. If, for example, guessed values of the future exogenous variables and expected values of the future error terms were used for the experiments, the results would not likely be much different from those presented in Tables 1 and 2, as long as the same set of values of the exogenous variables and error terms was used for all the experiments. For each experiment, the model would first be simulated using the particular set of values of the exogenous variables and error terms, and the predictions of the endogenous variables from this simulation would be recorded. The model would then be resimulated for the policy change, and the predictions from this simulation would be recorded. The numbers presented in the tables would then be the differ-

ences between the predicted values from the two simulations. Because the model is nonlinear, these numbers would not be the same as those presented in Tables 1 and 2, but they would probably be fairly close. The key assumption for Models 2 and 3 is that agents know the model and have the same expectations about the future exogenous variables and error terms, not that these expectations are necessarily perfect.

Second, it should be noted that the fact that equations (1)', (5), and (6) abstract from things like transactions costs, preferred habitats, and differential tax treatments is not likely to have much of an effect on the results in the two tables. These factors are probably fairly constant across time and across policy changes, and so their effects can fairly safely be assumed to be absorbed in the error terms.

VI. Conclusion

This study has demonstrated that it is feasible to analyze a large-scale macroeconometric model with rational expectations in the bond and stock markets. With some qualifications, the results indicate that fiscal-policy and monetary-policy actions are about half as effective when there are rational expectations in these markets than when there are not. One important qualification is that if the fiscal-policy action is accompanied by a monetary-policy action that keeps the bill rate unchanged, then it makes little difference whether or not the model is one with rational expectations in the bond market. The results also indicate, again with qualifications like the one just mentioned, that the existence of rational expectations in the bond market is quantitatively more important than is the existence of rational expectations in the stock market.

The second question posed at the beginning of this paper has thus

been answered for a particular model and set of variables. The properties of the model do appear to be sensitive enough to the assumption of rational expectations in the bond and stock markets to make the answer to the first question of some concern. To conclude this paper, a few remarks about a way in which one might test the assumption of rational expectations in bond and stock markets will be made.

If expectations of future bill rates and future values of after-tax cash flow are assumed to be perfect, then equations (1)' and (4)-(6) can be used to compute predicted values of RA, RM, and CG in the manner discussed in Section IV. This was done, and for the period 1954I-1968II, which ends 32 quarters before the end of the data period, the correlation between the actual and predicted values of RA is 0.878. This compares to a correlation of 0.993 for the predicted values of RA computed from equation (1). For RM, the correlation is 0.726 using the predicted values from equation (1)' and 0.988 using the predicted values from equation (2). For CG, the correlation is -0.140 using the predicted values from equations (4)-(6) and 0.111 using the predicted values from equation (3).

The "perfect foresight" predictions of RA, RM, and CG are thus not as accurate as the predictions computed from the more traditional equations. The perfect foresight assumption is, however, quite extreme, and the numbers just cited are not a test of the assumption of rational expectations. One possible test of this assumption is, however, the following: (1) Choose for each quarter a set of future values of the exogenous variables and error terms that one believes were expected at the time. (In most cases the future values of the error terms would be zero.) (2) Using Model 3, or a similar model, compute for each quarter the predicted values of RM, RA, and CG. The predictions of these variables for each quarter

would be based on different initial conditions and a different set of future values of the exogenous variables. (3) Compare these predicted values with the actual values. To the extent that the model is the one used by everyone and to the extent that the exogenous-variable values correctly reflect the expectations at the time, the predictions of RA, RM, and CG are the predictions implied by the rational expectations hypothesis. Their accuracy could thus be compared to the accuracy of other predictions as a test of the hypothesis. It is beyond the scope of the present study to attempt such a test.

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